

Assignment 1 (Solution Sheet)

This assignment is worth a total of 40 points.

1. [Topic: Basic. Level: 1.] $y = \frac{\sin x}{x}$. (5 points)

Solution:

$$\frac{dy}{dx} = \frac{\cos x}{x} - \frac{\sin x}{x^2}$$

2. [Topic: Basic. Level: 1.] $f(x) = g(x) \ln(g(x))$. (5 points)

Solution:

$$\begin{aligned} f'(x) &= g'(x) \ln(g(x)) + \frac{g(x)}{g(x)} g'(x) \\ &= g'(x)(1 + \ln(g(x))). \end{aligned}$$

3. [Topic: Basic. Level: 2.] $y = \tan x$ (10 points)

Solution:

$$\begin{aligned} y &= \tan x \\ &= \frac{\sin x}{\cos x} \\ \frac{dy}{dx} &= \frac{\cos x}{\cos x} + \sin x \times \frac{-1}{\cos^2 x} \times -\sin x \\ &= 1 + \tan^2 x \\ &= \sec^2 x. \end{aligned}$$

4. [Topic: Theory. Level: 3.] Differentiate from first principles $f(x) = \sqrt{x}$ (20 points)

Solution:

$$\begin{aligned} \frac{df}{dx} &= \lim_{\Delta x \rightarrow 0} \frac{\sqrt{x + \Delta x} - \sqrt{x}}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{(\sqrt{x + \Delta x} - \sqrt{x})(\sqrt{x + \Delta x} + \sqrt{x})}{\Delta x(\sqrt{x + \Delta x} + \sqrt{x})} \\ &= \lim_{\Delta x \rightarrow 0} \frac{x + \Delta x - x}{\Delta x(\sqrt{x + \Delta x} + \sqrt{x})} \\ &= \lim_{\Delta x \rightarrow 0} \frac{\Delta x}{\Delta x(\sqrt{x + \Delta x} + \sqrt{x})} \\ &= \lim_{\Delta x \rightarrow 0} \frac{1}{\sqrt{x + \Delta x} + \sqrt{x}} \\ &= \frac{1}{2\sqrt{x}} \end{aligned}$$