1 Differentiation Problems

1. $y = \tan x$
2. $f(x) = g(x)\ln(g(x))$.
3. $y = \arctan x = \tan^{-1} x$
4. Given

$$\lim_{x\to 0} \frac{\cos x - 1}{x} = 0$$
$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

differentiate from first principles $f(x) = \cos x$.

2 Solutions

1.

$$\begin{aligned} y &= \tan x \\ &= \frac{\sin x}{\cos x} \\ \frac{dy}{dx} &= \frac{\cos x}{\cos x} + \sin x \times \frac{-1}{\cos^2 x} \times -\sin x \\ &= 1 + \tan^2 x \\ &= \sec^2 x. \end{aligned}$$

2.

$$\begin{aligned} f'(x) &= g'(x)\ln(g(x)) + \frac{g(x)}{g(x)}g'(x) \\ &= g'(x)(1 + \ln(g(x))). \end{aligned}$$

3.

$$\tan y = x$$

diff w.r.t. x:

$$\begin{aligned} \sec^2 y \frac{dy}{dx} &= 1 \\ \frac{dy}{dx} &= \frac{1}{\sec^2 y} \\ &= \frac{1}{1 + \tan^2 y} \\ &= \frac{1}{1 + x^2} \end{aligned}$$

4.

$$\begin{aligned}
\frac{df}{dx} &= \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\cos(x+\Delta x) - \cos(x)}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\cos x \cos \Delta x - \sin x \sin \Delta x - \cos x}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\cos x(\cos \Delta x - 1) - \sin x \sin \Delta x}{\Delta x} \\
&= \cos x \lim_{\Delta x \to 0} \frac{\cos \Delta x - 1}{\Delta x} - \sin x \lim_{\Delta x \to 0} \frac{\sin \Delta x}{\Delta x} \\
&= -1 \qquad \text{(using given results)}
\end{aligned}$$