

## Assignment 1 (Solution Sheet)

1.  $f(x) = g(x)^{h(x)}$ .

**Solution:**

$$\begin{aligned} f(x) &= e^{\ln g(x)^{h(x)}} \\ &= e^{h(x) \ln g(x)} \\ f'(x) &= e^{h(x) \ln g(x)} \left( h'(x) \ln g(x) + h(x) \frac{g'(x)}{g(x)} \right) \\ &= g(x)^{h(x)} \left( h'(x) \ln g(x) + \frac{h(x)g'(x)}{g(x)} \right) \end{aligned}$$

2.  $y = \arcsin(x)$

**Solution:**

$$\sin(y) = x$$

diff. w.r.t.  $x$ :

$$\begin{aligned} \cos y \frac{dy}{dx} &= 1 \\ \frac{dy}{dx} &= \frac{1}{\cos y} \\ &= \frac{1}{\sqrt{1 - \sin^2 y}} \\ &= \frac{1}{\sqrt{1 - x^2}}. \end{aligned}$$

3.  $y = \arccos x$ .

**Solution:**  $\cos y = x$  diff. w.r.t.  $x$ :

$$\begin{aligned} -\sin y \frac{dy}{dx} &= 1 \\ \frac{dy}{dx} &= \frac{-1}{\sin y} \\ &= \frac{-1}{\sqrt{1 - \cos^2 y}} \\ &= \frac{-1}{\sqrt{1 - x^2}} \end{aligned}$$

4.  $y = \tan x$

**Solution:**

$$\begin{aligned}y &= \tan x \\ &= \frac{\sin x}{\cos x} \\ \frac{dy}{dx} &= \frac{\cos x}{\cos x} + \sin x \times \frac{-1}{\cos^2 x} \times -\sin x \\ &= 1 + \tan^2 x \\ &= \sec^2 x.\end{aligned}$$

5.  $y = \arctan x = \tan^{-1} x$

**Solution:**

$$\tan y = x$$

diff w.r.t.  $x$ :

$$\begin{aligned}\sec^2 y \frac{dy}{dx} &= 1 \\ \frac{dy}{dx} &= \frac{1}{\sec^2 y} \\ &= \frac{1}{1 + \tan^2 y} \\ &= \frac{1}{1 + x^2}\end{aligned}$$

6.  $y = (\tan x)^{-1} = \cot x$

**Solution:**

$$\begin{aligned}\frac{dy}{dx} &= -(\tan x)^{-2} \sec^2 x \\ &= -\frac{\cos^2 x}{\sin^2 x} \cdot \frac{1}{\cos^2 x} \\ &= \frac{-1}{\sin^2 x} \\ &= -\csc^2 x.\end{aligned}$$

7.  $y = \cos(x^2) \sin x$ .

**Solution:**

$$\frac{dy}{dx} = -\sin(x^2)2x \sin x + \cos(x^2) \cos x$$

8.  $y = (x + 1) \ln(x + 1)$ .

**Solution:**

$$\begin{aligned}\frac{dy}{dx} &= \ln(x + 1) + \frac{x + 1}{x + 1} \\ &= 1 + \ln(x + 1).\end{aligned}$$

9.  $f(x) = g(x) \ln(g(x))$ .

**Solution:**

$$\begin{aligned} f'(x) &= g'(x) \ln(g(x)) + \frac{g(x)}{g(x)} g'(x) \\ &= g'(x)(1 + \ln(g(x))). \end{aligned}$$

10.  $y = \frac{\sin x}{x}$ .

**Solution:**

$$\frac{dy}{dx} = \frac{\cos x}{x} - \frac{\sin x}{x^2}$$